Phys 455

Earthquake Victim Location Transponder

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Abstract

A transponder device has been proposed that would be worn by those in an earthquake prone region. The device emits an electromagnetic signal that would be detected by a rescue crew, thereby facilitating the recovery of victims buried under debris. Furthermore, the device has the ability to distinguish between those who are alive and those who have died, allowing for a triage approach to victim recovery. Though some parts of the device have been specified, other areas of design would require further research and testing before the device would be feasible to market.

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1.0 Introduction:

1.1 – What are transponders?

Transponders, also referred to as transceivers or beacons, are devices that emit an electromagnetic signal upon receiving an incoming signal, usually referred to as the interrogator signal. Examples vary from those devices that are employed on satellites to drive the telecommunications industry, to the type of transponder one uses on the fully automated 407 highway in Toronto. There are two types of transponders, being either active or passive.[4] Passive transponders are those that merely act as reflectors to the interrogator signal, whereas active transponders strengthen the interrogator signal and then transmit a signal back. The type of transponder being considered in this report is of the active type due to the high losses the signal would undergo while travelling through debris.

1.2 – Design goals

The basic problem at hand is to design a device that, in the event of an earthquake, emits an electromagnetic signal that will facilitate the location of a person buried under rubble. The basic goals that need to be met for this device are:

- the device must be able to send a strong enough signal through the debris to be detected by the rescue team
- the device must be small enough and light enough to be carried comfortably by the wearer

- the device should distinguish between those still alive and those who have died
- the cost of the device should be minimized so as to be affordable by all

1.3 – Existing devices

Today, there are hundreds of transponders on the market with as many different applications. However, one particular transponder that fulfills similar requirements to those being sought is the Tracker DTS manufactured by Backcountry Access, Inc. This transponder is used by skiers in the event of being buried by an avalanche. This device runs on three AAA alkaline batteries, operates at 457 kHz, weighs 298 g, and can transmit for about 250 hours through up to 50 m of snow.[9] This device cannot be activated remotely, and electromagnetic radiation is attenuated less in snow than in concrete (see 2.2 below), but nevertheless is a good example of an existing device that comes close to satisfying the design requirements outlined in this report.

1.4 – Importance of such a device

Such a device would be of great importance in earthquake prone regions since it would speed up the recovery time of buried victims in a situation where time is of the essence. Currently, many people resort to using their cell phones in disaster situations, if they have one. The advantage of the device proposed here is that it will be better equipped to transmit through large amounts of debris that would be typical in the event of an earthquake, due to its lower operating frequency. Also, even though someone may have a cell phone with them, they may lose it during an earthquake, or they may be unable to access it on account of the rubble they are trapped under. The proposed device would circumvent the latter of these two circumstances by having the option of being triggered remotely, and with regards to the former situation, the wearer would be less likely to lose the device since it would be constantly attached to their person.

2.0 Theory:

2.1 – Electromagnetic radiation

The basic phenomenon being exploited in the operation of this device is that of electromagnetic radiation. Though the basic laws describing electromagnetic waves are Maxwell's equations, we are more interested in the form these equations take when describing the electric and magnetic fields for an antenna. A basic model for describing fields of this type is called the Hertzian dipole. In this model we imagine a current flowing between two small charged spheres. This model leads to Figure 1 below:

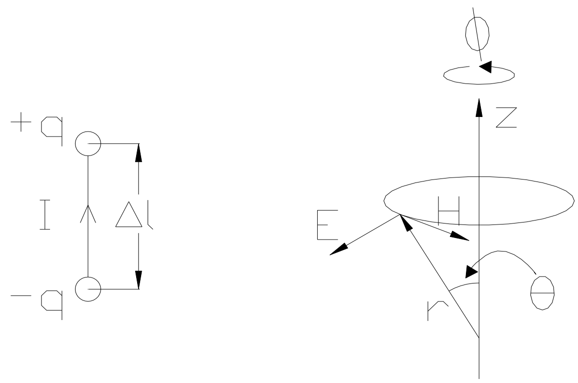

Figure 1 – Hertzian dipole and the relative directions of its fields

Where the electric field has components in the θ and r hat directions, while the magnetic field is defined entirely in the ϕ hat direction. Thus, the mathematical representation of these fields is as follows:

$$\vec{H}(\vec{r}) = \frac{\omega k}{4\pi}\frac{e^{ikr}}{r}(1-\frac{1}{ikr})\hat{r}\times\vec{p} \quad (1a)$$ [20]

Where ω is the angular frequency, k is the wave number (defined below), and p is the dipole moment, defined to be equal to qΔl as shown in Figure 1. But as we have already stated, the H-field is entirely in the ϕ hat direction; this follows since r x p=-(p x sinθ) [ϕ hat]. So we can rewrite this as:

$$H_{\phi} = \omega kp\sin\theta\frac{e^{ikr}}{4\pi r}(\frac{1}{ikr}-1) \quad (1b)$$

Notice that the H-field is composed of two components; one proportional to r^{-1}, called the radiation field, and one proportional to r^{-2}. This will become important later when we attempt to calculate the power radiated by the source. The electric field has components given by:

$$E_r = \frac{p\cos\theta}{2\pi\varepsilon}\frac{ike^{ikr}}{r^2}\left(\frac{1}{ikr}-1\right) \qquad (2a)$$

$$E_\theta = -\frac{k^2 p\sin\theta}{4\pi\varepsilon}\frac{e^{ikr}}{r}\left(1-\frac{1}{ikr}+\frac{1}{(ikr)^2}\right) \qquad (2b)$$

Where ε is permittivity of the material. Figure 2 is a diagram of what these electric fields look like:

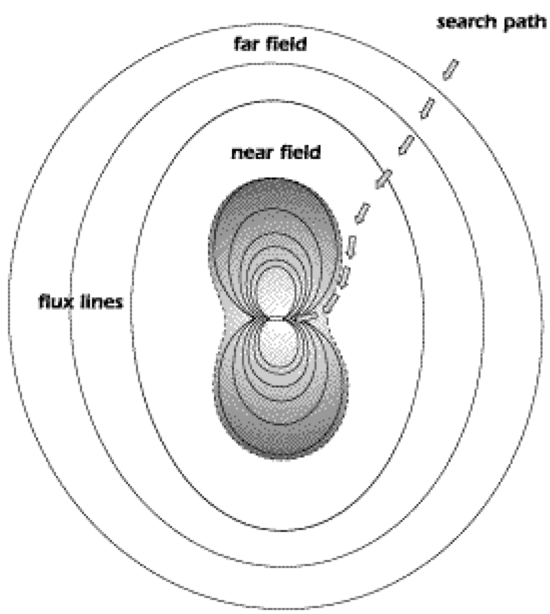

Figure 2 – Electric field lines [9]

Notice again that the electric field is a polynomial where the term proportional to r^{-1} dominates the far field, or radiation field, whereas the term proportional to r^{-3} dominates in the near field region. In determining the energy radiated from the source we care only about the radiation components of the above fields. From these equations it can be shown that the amplitude of the radiation component of the electric field is given by:

$$E_0 = \frac{H_{rad}\eta r}{\sin\theta} e^{-ikr} \quad (3)$$ [20]

Where η is the intrinsic impedance of the material. Now, the time averaged power radiated is given by:

$$\overline{P} = \frac{4\pi}{3}\frac{|E_0|^2}{\eta} \quad (4)$$ [20]

Notice that the power radiated is only dependent on the radiation component of the electric field. This last equation will assist in determining the amount of power that must be provided by the power source. The time averaged power can be related to the current, I, flowing through the dipole by the equation:

$$\overline{P} = \frac{\pi\eta}{3}\left(\frac{\Delta l}{\lambda}\right)^2 |I|^2 \quad (5)$$ [20]

Where Δl is the length of the dipole, or in our case the length of the antenna through which the current flows, and λ is the wavelength. One other equation that we will need to make use of is the wave equation:

$$k = \frac{2\pi n}{\lambda} = \frac{2\pi \nu n}{c} \quad (6)$$ [6]

Where k is called the wave number, n is the index of refraction, ν is the frequency, and c is the speed of the wave.

2.2 – Skin depth

The skin depth of a material is a measure of the distance in a particular material required for an electromagnetic wave to be attenuated to 1/e of its initial value. The skin depth is given by:

$$\delta = \frac{1}{\sqrt{\pi \nu \mu \sigma}} \quad (5)$$

[5]

Where μ is the permeability of the material and σ is the conductivity. As mentioned above, one of the considerations of this device is that electromagnetic signals are attenuated more in concrete than in snow. Thus, this device will have to have a stronger power supply than the Tracker DTS. Equation 3 shows why. If we take the permeability of both concrete and snow to be approximately equal to the permeability of free space μ_0, and consider that both devices have the same operating frequencies (as will be specified later), then it is easy to see that the only difference in the materials' skin depth comes from their conductivity. Snow has a conductivity on the order of 10^{-4} or smaller[5], while concrete has a conductivity on the order of 10^{-3}.[2] Thus, concrete has a smaller skin depth and attenuates signals more quickly than does snow.

3.0 Proposed Solution:

3.1 – Operating frequency

Choosing the frequency that the device will operate at is an important parameter to set. As mentioned above, the skin depth of a material is inversely proportional to frequency. Cell phones typically operate in the upper radio frequency range, ~800 MHz.[13] Thus, using Equation 5, a cell phone's signal would have a skin depth in concrete of about:

$$\delta = \frac{1}{\sqrt{\pi \times 800 \times 10^{6} \times \mu_{0} \times 10^{-3}}} \cong 0.56m$$

The governing body that sets the standards for the telecommunications industry in Europe is the European Telecommunication Standard, or ETS. The ETS code that covers the standards that would apply in the design of this device is ETS 300 718: Radio Equipment and Systems; Avalanche Beacons; Transmitter-receiver Systems. This standard lists frequencies that are reserved for emergency situations. One of these frequencies, 457 kHz, is the frequency that has been selected for this device to operate at.[3] Considering the skin depth in concrete at this frequency:

$$\delta = \frac{1}{\sqrt{\pi \times 457 \times 10^{3} \times \mu_{0} \times 10^{-3}}} \cong 23.54m$$

Thus, the operating frequency that has been selected conforms to ETS 300 718, and has a significantly lower skin depth than a cell phone signal typically would. Also, the American analog of this standard, ASTM F1491-93, set by the American Society for Testing and Materials, only sets 457 kHz as a useable frequency.[10] Thus choosing 457 kHz as the operating frequency gives a greater marketing scope to the device.

3.2 – Thermometer

In order for this device to distinguish between those wearers who are still alive and those who have died, it is proposed that a digital thermometer be used to monitor body temperature. The output from the thermometer could then be used to emit different signals, which would indicate the state of the victim. How is this possible?

3.2.1 – Body temperature

What makes this possible, is the fact that the human body normally maintains a fairly steady body temperature of about 37°C.[14] Of course, there is a small range that each individual's body temperature will lie in, due to varying metabolic rates, lifestyles, and environmental conditions. However, even after taking these factors into account, over 95% of people have a body temperature that falls within 36.4°C to 37.2°C.[14] Other fluctuations in body temperature that occur simply to physical exercise or the time of day, still only amount to a difference of about half a degree to a degree of what is normal for that individual. All told, most people's normal body temperature will remain in a range of about 35.4°C to 38.2°C, regardless of their metabolic rate, what they are doing, or what time of day it is. Having said this, effects of hypothermia begin once body temperature drops below 35°C, and a person will fall into a coma at 32.2°C. Death occurs at a temperature of 25.6°C.[15] Once death occurs, the body's temperature will decrease at a rate of about one degree per hour, until it reaches the temperature of its environment.[8] With these numbers in mind, it is proposed that once the body temperature of the wearer falls below 32°C, that the thermometer monitoring body temperature begin to send a different signal to the radiating antenna. In this way, a rescue team could apply a triage approach to saving victims by looking first for those who have a greater chance of survival.

Using a conventional mercury filled thermometer, the best places to take a person's temperature are under the arm, in the mouth, the rectum, and in the ear.[11] Obviously, none of these methods for measuring body temperature are practical for the type of device being proposed, since the wearer would be carrying the device at all times. A company has been found that specializes in non-invasive thermometry systems called Exergen Corporation. One of the products they manufacture is called the DermaTemp, which measures skin temperature using an infrared thermographic scanner to detect tiny variations in perfusion.[12] The problem with this existing device is that it is too big to be incorporated into a design where someone carries it all day. However, this type of thermometer could be made smaller to be used in a portable device. Some of the properties of the existing device are that it has an accuracy of ±0.1°C, a response time of 0.1 s, and has a temperature range of 18°C to 43°C. This particular model is made for industrial duty applications with an impact resistant casing. This is part of the reason for its large size and weight: 9cmx18cmx2cm and 255 g.[12] This model also has a large LED display that would be unnecessary for the purposes of the proposed device. The DermaTemp runs off a standard 9 V alkaline battery and can take approximately 5 000 readings. Thus, the power requirements and operating lifetime are well within range of what is needed for the proposed device, especially considering that the DermaTemp is made for more rugged use. For the thermometer part of the proposed device, more time and research would need to be done to ensure that this type of technology could be made small enough.

3.3 – Power source

In section 3.1 the operating frequency was set in accordance with ETS 300 718. ETS 300 718 also sets the standard for the minimum H-field strength at a specified distance. According to ETS 300 718, the minimum H-field strength shall be no lower than 0.5 μA/m at a distance of 10 m. Recall from Figure 1 that θ is measured from the z-axis to the r vector. If we refer back to Figure 2, which shows a diagram of the electric field pattern with z hat running from top to bottom of the diagram, we can see that the minimum electric field strength is going to occur when θ is equal to 90°. Since the electric field and H-field are codependent, the H-field will also be a minimum when θ is 90°. With this information in mind we set out to calculate the power emitted by the source. Recall that Equation 4 is only dependent on the radiation components on the fields emitted by the source. The relationship between the total H-field and radiation component of the H-field is a simple proportionality constant obtained from Equation 1b:

$$H_{\phi} = H_{tot} = H_{rad}(\frac{1}{ikr} - 1)$$

so that:

$$H_{rad} = \frac{H_{tot}}{(\frac{1}{ikr} - 1)}$$

Subbing this form Equation 1b into Equation 3 gives:

$$E_0 = \frac{H_{tot}}{(\frac{1}{ikr} - 1)} \frac{\eta r}{\sin\theta} e^{-ikr}$$

Before we sub the above result into Equation 4 for the power, we need to take the square of the absolute value of E_0. It should be noted that $|E_0|^2$ really means: $E_0 x E_0^*$, where E_0^* is the complex conjugate of E_0. Following through with this yields:

$$|E_0|^2 = \frac{|H_{tot}|^2 \eta^2 r^2}{\left(\frac{1}{(kr)^2} + 1\right) \sin^2\theta}$$

Which, when subbed into Equation 4 produces:

$$\overline{P} = \frac{4\pi}{3} \frac{|H_{tot}|^2 \eta r^2}{\left(\frac{1}{(kr)^2} + 1\right) \sin^2\theta}$$

Now, $k=2\pi\nu n/c$, but $n=\sqrt{\mu_r \varepsilon_r}$ for the material in question. We already specified the frequency in Section 3.1 to be 457 kHz. The relative permeability of concrete, μ_r, is taken to be 1, while the relative permittivity, ε_r is 6.[18] Making the appropriate substitutions we find $k=2\pi x 457x10^3 x\sqrt{6}/3x10^8=0.0235$ m^{-1}. Also, the intrinsic impedance is given by $\eta=\sqrt{\mu_r \mu_0/\varepsilon_r \varepsilon_0}$.[5] Therefore, $\eta=\sqrt{1 x \mu_0/6 x \varepsilon_0}=153.8\ \Omega$. Finally, substituting in all of the above values, including those specified in ETS 300 718, gives:

$$\overline{P} = \frac{4\pi}{3} \frac{\left|0.5\times10^{-6}\right|^{2} 153.8\times10^{2}}{\left(\frac{1}{(0.0235\times10)^{2}}+1\right)\sin^{2}90} = 1.363\times10^{-8}\ \text{W}$$

This is the minimum amount of power the source needs to produce. With this result we can proceed to calculate the current that would flow through the antenna using Equation 5:

$$I = \sqrt{\overline{P}\frac{3}{\pi\eta}}\left(\frac{\lambda}{\Delta l}\right)$$

The wavelength will be $\lambda = c/n\nu = 3\text{x}10^{8}/(\sqrt{6}\text{x}457\text{x}10^{3}) = 267.8$ m. This gives a current of:

$$I = \sqrt{1.3636\times10^{-8}\frac{3}{\pi 153.8}}\left(\frac{267.8}{\Delta l}\right) = \frac{2.464\times10^{-3}}{\Delta l}\ \text{A}$$

All that remains is for us to specify the length of the antenna. Given that we want this device to be reasonably small, it is proposed that the emitting antenna be 2 cm, as a working number. Substituting this value into the above result gives a current of 0.123 A. Finally, we can use this value to suggest a suitable power source. A company called Matsushita manufactures a relatively new type of lithium polymer battery, model number SSP356236. The properties of this battery "makes it ideal for compact portable electronic equipment".[17] This battery has dimensions of 35mmx62mmx3.6mm, and weighs only 15 g.[17] It can operate in temperatures ranging from -10°C to 60°C, which definitely makes it applicable for the proposed device. Also, it has an average electric capacity of 0.5 Ah. Now, if we divide this capacity by the current we calculated above, we find that one such battery could operate the proposed device for just over 4 hours. Based upon wearer needs, it is generally held that a person cannot survive for more than about three days without water.[16] Thus, if we want to design this device to last for three days, then almost 18 lithium polymer batteries would be needed. This would put the total weight of the power supply at about 266 g, and give it dimensions of about 35mmx62mmx64mm if stacked along its thinnest dimension. Thus, in order to fulfill wearer needs, a rather bulky power supply is in order, even while using some of the latest technology.

3.4 – Other components

There are a few other key components that need to be specified in order to make this device operable. A simple wireless modem needs to be found, or developed, to select distinguishable transmission signals based upon the output of the digital thermometer. The existing wireless technology that would be suited for handling this type of task is vastly overqualified. As an example, Dell produces a wireless modem called ‘Go America’, manufacturer part# MINS12MM.[1] This product is 66mmx11mmx48mm and weighs 100 g. It can operate in temperatures ranging from 0°C to 45°C, which would be suitable considering that the wearer would be carrying the device close to their body. Also, it runs off a single rechargeable lithium ion battery. Of course, when properly integrated into the proposed device this component could be made much smaller and lighter than a product like Go America, since it does not need to transmit the type of data typically sent by a wireless modem hooked up to laptop or PC. The only function the modem for the proposed device would serve is to take input from the thermometer, and output two distinguishable signals based upon it. Again, these distinguishable signals would indicate whether the victim was alive or dead. One method of accomplishing this variation in signals would be to vary their frequency. For example, the signal indicating someone who is still alive would be sent at 457.05 kHz, whereas the signal sent for someone who has died be sent at 456.95 kHz. Though nothing is mentioned about the required frequency sensitivity of the receiver in ETS 300 718, it is the author's general impression, that receivers have a fairly high frequency sensitivity and that there is a small range about the allotted 457 kHz that is reserved for emergency situations. More research would need to be done to make this point conclusive. Perhaps a better possibility for discerning between signals could be based upon field strength output. In keeping with the standards set by ETS 300 718, the receiver would have to have a minimum signal to noise ratio of 6 dB when the field strength is 80 nA/m.[3] More time would need to be spent on determining other possible solutions.

One other component that would need to developed, though the author makes absolutely no claims as to how this would be designed, is a microcircuit to connect all these components together. The only reason it is mentioned is the realization that almost all electronics today are organized by a microcircuit.

3.5 – Assembly

The basic design consists of a power supply, a transmitting and receiving antenna, a digital thermometer to monitor the wearer's body temperature, and a modem to characterize different signals based upon the output from the thermometer. The device will have four modes it can be in. The first mode is simply when the device is turned off. This will keep the device from drawing power when it is not needed. When the wearer turns the power on, the device will immediately be in a standby mode. In standby mode the device will have the ability to be switched to transmit mode manually and remotely. Allowing the device to be triggered to transmit mode remotely has the advantage of being able to locate those people who cannot switch their transponder to transmit mode because of debris they are buried under. The last two modes are the transmitting modes, again one mode for those who are alive and one for those who are not. A basic schematic of the device is shown in Figure 3 below:

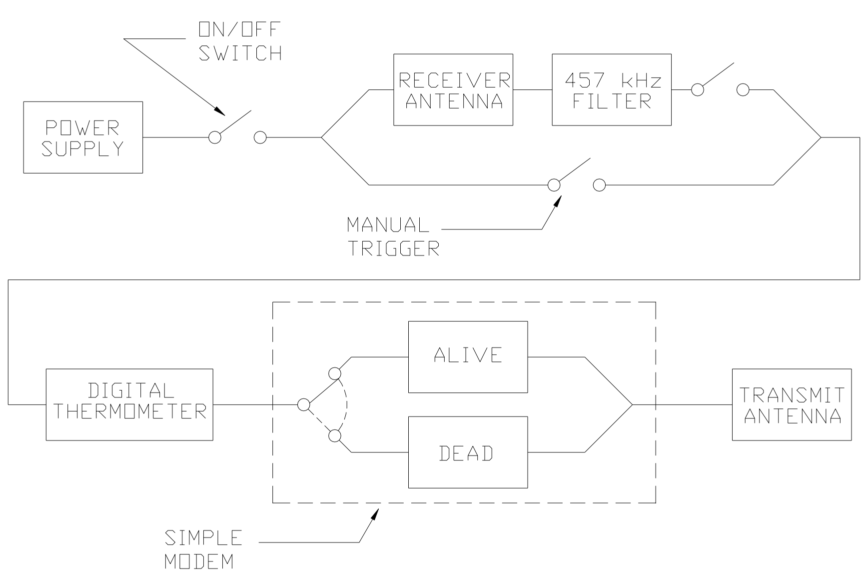

Figure 3 – Schematic of an earthquake victim location transponder

Notice that a 457 kHz filter is also necessary in order to sift out all other electromagnetic radiation. Another point to be made is that the antennas would probably be loop antennas wound around a ferrite core in order to beef up any incoming and outgoing signals. Once all these components are assembled, a hard plastic casing would seal the components inside, except for the probe from the thermometer. A clip would also be necessary since it is expected that the device would be clipped to the waist line of the wearer's pants. This way, the device would be worn on the outside of the pants with the temperature probe tucked under the pants to monitor the wearer's body temperature.

The total weight of the device using existing products would be about: 255 g (thermometer) + 100 g (modem) + 15x18 g (batteries) + ~50 g (casing, antennae, and other) ≈ 675 g. This compares to 298 g for the Tracker DTS, and about 164 g for the average cell phone.[19] Total size would be about: 324 cm^3 (thermometer) + 35 cm^3 (modem) + 7.8x18 cm^3 (batteries) + ~20 cm^3 ≈ 520 cm^3.

4.0 Cost Analysis:

Table 1 below is a summary of the projected cost of one unit based upon the prices of existing products:

Table 1: Cost analysis of an earthquake victim location transponder

Component	**Cost***
Power supply**	~$60x18=$1 080
Thermometer**	~$350

Simple modem	$165
Antennae	a few dollars
Casing	a few dollars
Total	**$1 600**

*All prices are in US dollars

**Cost of product was obtained by talking to a sales representative from the respective company

The total cost of this product is outrageously expensive, especially when you consider that the Tracker DTS is priced at $300 US. Still, it is posited that with further research, and parts integration, the cost of such a device could be dramatically reduced. Furthermore, if this device were considered highly desirable by a government in an earthquake prone region, it is conceivable that such a government would subsidize part of the price.

5.0 Future Considerations:

Though some preliminary research has been done to design an earthquake victim location transponder, obviously there are many other considerations that need to be taken. In terms of the signals radiated by the device, concrete was used as the model for the debris that someone might be buried under. In actuality, someone would very likely be buried under a whole host of other materials, such as brick, metal, and glass, each with their own dielectric properties. In order to figure out how much radiation could actually penetrate a composition of materials, empirical data based upon an array of tests would likely be the most accurate method. Also, the end result of the power calculations resulted in a source that was far too large to be carried by the wearer. One attractive possibility that circumvents this problem without compromising the victims' needs, is to design the device to transmit an intermittent signal rather than a continuous one. In this way, less power would be needed, and the source could be made much smaller. One other note about the power supply is that the Hertzian dipole model used to describe the source may not be accurate enough. In this case, a more in depth approach that includes a current distribution along the antenna would need to be taken.

The current technology for thermometers, and even more so for wireless modems, seems to indicate that the type of digital thermometer and signal selector required for this device to operate is certainly attainable. Yet, at present none of the devices manufactured from those existing technologies quite achieve the objectives needed for this to device to work. The DermaTemp mentioned in Section 3.2 is far too big to be of use, and more research would need to be done in order to minimize its size. Also, this device is designed to distinguish between wearer's based upon their body temperature; one signal for below 32°C and another signal for above 32°C. Therefore, if this device was being used in a hot environment where the ambient temperature was greater than or equal to 32°C, this device would fail to distinguish between victims.

The length of the antenna that transmits the rescue signal would also need to be optimized. It was suggested in this report that a ferrite-core, wire wound antenna be used, though this was merely based upon the design of the existing Tracker DTS device.[7]

6.0 Conclusions and Recommendations:

An outline for an earthquake victim location transponder has been proposed. The basic design consists of a power supply, transmission and receiver antennas, a digital thermometer, and a simple modem. Based upon calculations, it seems reasonable to create a power supply that could emit a signal strong enough to be detected by a rescue team. Implicit in this is that the receiver used by the rescue team should conform to those standards set for receivers in ETS 300 718. Following some further research, an optimized solution for creating distinguishable output signals could be attained. However, the total weight and volume of the device using existing products is about 675 g and 520 cm^3. The total cost of one unit being roughly $1 600 US, or about $2 560 Canadian. This seems a rather cumbersome device for someone to carry around all day and far too expensive. However, once all the components are optimized for their intended purpose, and integrated into one circuit, it is entirely likely that the overall weight and price would be significantly reduced. Finally, it can be said that the first and third of the design goals have been met, the second and fourth, are still a work in progress, and thus require more time to achieve.

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